The 7th GraphMasters Workshop

Lleida, June 25–28, 2013

ABSTRACT BOOK

Cryptography & Graphs
Research Group

Universitat de Lleida. Catalonia

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Contents

Welcome

It is a pleasure to welcome you to the 7th GraphMasters workshop, which is being held at the Escola Politècnica Superior in Lleida, organized with the support of the Mathematics Department and the InsPIReS Research Center of Universitat de Lleida.

The GraphMasters workshop is a series of informal meetings of small groups of researchers working on various recent developments and open problems in Graph Theory. A particular GraphMasters meeting is usually concerned with just a small number of related open problems. This seventh edition is focused on several aspects of the degree/diameter problem.

The former GraphMasters workshops editions took place in ITB Bandung, Indonesia (December 2010, July 2012), Newcastle, Australia (February 2012, December 2012), Lleida, Catalonia (September–October 2012), Shanghai, China (January 2013).

We would like to thank all the participants for sharing experiences, techniques and ideas which will allow to strengthen the bonds of friendship and collaboration among all of us. Special thanks to Mirka Miller and Joe Ryan for suggesting us to organize this GraphMasters workshop in Lleida.

We wish you a very fruitful workshop, as well as a very enjoyable stay in Lleida.

Cryptography and Graphs Research Group (C&G)
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Program

Tuesday, June 25, 2013

10:10–10:15 **Opening**

10:15–11:45 **Session 1**

Mirka Miller, The order/degree optimization problem

Josep M. Miret, Nonexistence of the almost Moore digraphs under the cyclotomic conjecture

11:45–12:15 **Coffee time**

12:15–13:00 **Session 2**

Ramiro Feria–Purón, Constructions of large graphs on surfaces

13:00–16:00 **Lunch time**

16:00–17:30 **Session 3**

Hebert Pérez–Rosés, Recent advances in the Degree/Diameter Subgraph Problem in the Mesh

Joe Ryan, MaxDDBS for digraphs

All sessions will be held in Room 1.04 of the Polytechnic School

Wednesday, June 26, 2013

10:15–11:45 **Session 4**

Miquel Àngel Fiol, On some spectral and quasi–spectral characterizations of distance–regular graphs

Cristina Dalfó, The (Δ, D) and (Δ, N) problems in double–step digraphs with unilateral distance

11:45–12:15 **Coffee time**

12:15–13:00 **Session 5**

Nacho López, Algorithmic and algebraic methods on Mixed Moore grahs

13:00–16:00 **Lunch time**

16:00–16:45 **Session 6**

Rinovia Simanjuntak, Metric dimension of graphs and digraphs: recent results and open problems

16:45–17:30 **Working Session**

All sessions will be held in Room 1.04 of the Polytechnic School

Thursday, June 27, 2013

10:15–11:45 **Session 7**

Dominique Buset, What's new about antimagic graphs

Elgin Kiliç, On Irregular Total Labellins of Graphs

11:45–12:15 **Coffee time**

12:15–13:00 **Session 8**

Charles Delorme, Large bipartite abelian Cayley graphs

13:00–16:00 **Lunch time**

16:00–17:30 **Session 9**

M. Camino Balbuena, An overview on $\{\mathbf{C_3}, \cdots, \mathbf{C_s}\}$ extremal graphs

Susana–Clara López, The $\otimes_h$–product: a link among labelings

All sessions will be held in Room 1.04 of the Polytechnic School

Friday, June 28, 2013

10:15–11:45 **Session 10**

Francesc Sebé, On the characterization of pagerank uniform digraphs

Hebert Pérez–Rosés, Endorsement Deduction and Ranking in Social Networks

11:45–12:15 **Coffee time**

12:15–13:00 **Session 11**

Joan Vilaltella, Some results on the structure of multipoles

13:00–13:15 **Closing remarks**

All sessions will be held in Room 1.04 of the Polytechnic School

On Some Spectral and Quasi–Spectral Characterizations of Distance–Regular Graphs[1]

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This is a new contribution to the question: Can we see from the spectrum of a graph whether it is distance-regular? By generalizing some results of Van Dam and Haemers, among others, we give some new spectral or quasi-spectral characterizations of distance-regularity. In particular, we present some results assuring that a graph G is distance-regular without requiring, as it is common in this area, that G is cospectral with a (feasible) distance-regular graph satisfying some combinatorial conditions. With this aim, and in addition to the spectrum, we use conditions on the girth and some other metric parameters of G, mainly related to its intersection and preintersection numbers.

Keywords Distance-regular graph, Spectral excess theorem, Local spectrum, Orthogonal polynomials, Preintersection numbers.

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An overview on $\{\mathbf{C_3}, \ldots, \mathbf{C_s}\}$–free extremal graphs

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For integers $s \geq 4$ and $n \geq s+1$, let $ex(n; \{C_3, \ldots, C_s\})$ denote the maximum number of edges in a graph on n vertices and girth at least $s+1$. We refer to it as the extremal function. By $EX(n; \{C_3, \ldots, C_s\})$ we denote the set of all simple graphs of order n, girth at least $s+1$ and with $ex(n; \{C_3, \ldots, C_s\})$ edges. A graph $G \in EX(n; \{C_3, \ldots, C_s\})$ is called a extremal graph. Graphs constructed by researches interested in the Cage Problem, provide good constructive lower bounds for the extremal number. In this talk we will see some exact values of the extremal function for $s = 4, 5, 6, 7, 10, 11$ and also some lower bounds. Moreover, we will revise the last results on the lower bounds on the order of a extremal graph guaranteeing that the girth is equal to $s+1$ and other structural properties.

Some of the references used in this talk are listed below.

Keywords Extremal graphs, girth

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What's new about antimagic graphs

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Let us consider a finite, simple, undirected and connected graph $\Gamma = (V, E)$ where $V = V(\Gamma)$ (respectively $E = E(\Gamma)$) is the set of vertices (respectively the set of edges) of Γ.
If e is the number of edges of Γ, an **antimagic labeling of Γ** is a bijective mapping

$$\begin{array}{ccc} E & \xrightarrow{\sigma} & \{1, 2, \ldots, e\} \\ \{u, v\} & \longmapsto & \sigma(\{u, v\}) = \sigma(uv) \end{array}$$

such that $w(v_1) \neq w(v_2)$ for every distinct vertices $v_1, v_2 \in V$ where $w(v)$ is **the weight of the vertex** v i.e. the sum of the labels of the edges incident to v in Γ. We call the label of an edge $\{u, v\} = uv$ of Γ the integer $\sigma(uv)$. A graph which has an antimagic labeling is called a **antimagic graph**.
In 1990, Hartsfield and Ringel conjectured that every connected graph other than K_2 has an antimagic labeling. In this talk we will give some recent progresses in this field of interest.

Keywords Antimagic graph labeling

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Nonexistence of almost Moore digraphs under the cyclotomic conjecture[1]

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Almost Moore digraphs of degree d and diameter k, in short (d, k)-digraphs, appear in the context of the *degree/diameter problem* as a class of extremal directed graphs, in the sense that their order is one less than the unattainable Moore bound $M(d, k) = 1 + d + \cdots + d^k$, where $d > 1$ and $k > 1$ denote the maximum out–degree and diameter, respectively. The adjacency matrix of a (d, k)-digraphs fulfills the matrix equations $AJ = dJ$ and $I + A + A^2 + \cdots + A^k = J + P$, where J denotes the all-one matrix and P is a *permutation matrix*. So far, the problem of their existence has only been solved when $d = 2, 3$ or $k = 2, 3, 4$.

In this talk we will show the nonexistence of (d, k)-digraphs with $k > 4$ and $d > 3$, under the assumption of the cyclotomic conjecture, which is related to the factorization of the polynomials $\Phi_n(1 + x + x^2 + x^3 + \cdots + x^k)$, where $\Phi_n(x)$ denotes the nth cyclotomic polynomial.

This is a joint work with Josep González and Mirka Miller, and it includes Joan Gimbert's contributions.

Keywords Almost Moore digraphs, degree/diameter problem.

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The (Δ, D) and (Δ, N) Problems in Double–Step Digraphs with Unilateral Distance

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We study the (Δ, D) and (Δ, N) problems for double-step digraphs considering the unilateral distance, which is the minimum between the distance in the digraph and the distance in its converse digraph, obtained by changing the directions of all the arcs.
The first problem consists of maximizing the number of vertices N of a digraph, given the maximum degree Δ and the unilateral diameter D^*, whereas the second one (somehow dual of the first) consists of minimizing the unilateral diameter given the maximum degree and the number of vertices. We solve the first problem for every value of the unilateral diameter and the second one for some infinitely many values of the number of vertices. Moreover, we compute the mean unilateral distance of the digraphs in the families considered.

Keywords Double-step digraph, unilateral distance, (Δ, D) problem, (Δ, N) problem.

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Large bipartite abelian Cayley graphs

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Dougherty and Faber [1] have already studied large abelian Cayley graphs. We concentrate here on bipartite ones, and take notice when possible of possible symmetry of the graphs.
Simple upper bounds for order of bipartite abelian Cayley of diameter d are $a_{i,d-1} + a_{i-1,d-1}$ for degree $2i$, and $2a_{i,d-1}$ for degree $2i+1$, where $a_{i,d}$ is the natural upper bound for abelian Cayley graphs of degree $2i$ and diameter d, given by $a_{0,i} = a_{i,0} = 1$ for $i \geq 0$ and $a_{i+1,d+1} = a_{i+1,d} + a_{i,d+1} + a_{i,d}$.
We have, like [1] lower bounds: if $2d = \sum_{j=1}^{i} a_j$ then $n \geq 2^{i-1} \prod_{j=1}^{i} a_j$. (without the bipartition constraint the sum can be replaced by $2d+1$).
At last the maximum order for bipartite Cayley graphs of degree $2i+1$ and diameter $i+1$ is twice the maximum order for Cayley graphs of degree $2i$ and diameter i.

Keywords Large graphs, Cayley graphs, abelian groups.

References

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Constructions of large graphs on surfaces

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We consider the degree/diameter problem for graphs embedded in a surface, namely, given a surface Σ and integers Δ and k, determine the maximum order $N(\Delta, k, \Sigma)$ of a graph embeddable in Σ with maximum degree Δ and diameter k. We introduce a number of constructions which produce many new largest known planar and toroidal graphs. We record all these graphs in the available tables of largest known graphs.
Given a surface Σ of Euler genus g and an odd diameter k, the current best asymptotic lower bound for $N(\Delta, k, \Sigma)$ is given by

$$\sqrt{\frac{3}{8}g}\Delta^{\lfloor k/2 \rfloor}.$$

Our constructions produce new graphs of order

$$\begin{aligned} &6\Delta^{\lfloor k/2 \rfloor} \quad \text{if } \Sigma \text{ is the Klein bottle, or} \\ &\left(\frac{7}{2} + \sqrt{6g + \frac{1}{4}}\right)\Delta^{\lfloor k/2 \rfloor} \quad \text{otherwise,} \end{aligned}$$

thus improving the former value by a factor of 4.

Keywords Degree–Diameter Problem, surfaces

Some results on the structure of multipoles[1]

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A cubic graph can be subdivided by an edge cut into parts called multipoles. As a result of the cut, a multipole has half-edges with one free end. An isolated edge with both ends free can be viewed as a multipole, too. A multipole need not be connected. We call m-pole a multipole with m free ends. According to the *Parity Lemma*, if a multipole can be 3-edge-colored then the number of free ends of each color must have the same parity as m. Therefore, the colorings of the free ends are not arbitrary. We refer to such colorings as states. Multipoles and their states are used to study non-3-edge-colorable cubic graphs, called *snarks* ([1], [2], [3]). A multipole with all states permitted by the Parity Lemma is known as a color-complete multipole, and a multipole that has at least one state in common with any other multipole is known as a color-closed multipole. In [4], the question is posed whether non-color-complete color-closed multipoles exist for $m \geq 5$. We affirmatively answer this question for $m \in \{5, 6\}$ and explain other results on the states and structure of multipoles, in particular of color-complete multipoles.

Keywords Cubic graph, multipole, edge-coloring, Parity Lemma, snark, color-complete multipole, color-closed multipole.

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Open Problems in Irregular Total Labellings of Graphs

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A labelling of a graph is a map that carries graph elements to the numbers. The most common choices of domain are the set of all vertices (*vertex* labellings), the edge set alone (*edge* labellings), or the set of all vertices and edges (*total* labellings).
Chartrand, Jacobson, Lehel, Oellermann, Ruiz and Saba [2] proposed the following problem: Assign positive integer labels to the edges of a simple connected graph of order at least 3 in such a way that the graph becomes irregular, i.e., the weights (label sums) at each vertex are distinct. What is the minimum value of the largest label over all such irregular assignments? This parameter of a graph G is known as the *irregularity strength* of the graph G, $s(G)$.
Baca, Jendrol, Miller and Ryan [1] introduced a new idea of irregular total labelling as: For a graph $G = (V, E)$ with vertex set V and edge set E a labelling $\delta : V \cup E \longrightarrow \{1, 2, \ldots, k\}$ is said to be a total k–labelling. A total k–labelling is an *edge irregular total k–labelling* of G if for every two different edges e and f of G,

$$wt(e) \neq wt(f);$$

it is a *vertex irregular total k–labelling* of G if for every two distinct vertices x and y of G

$$wt(x) \neq wt(y).$$

The minimum k for which the graph G has an edge irregular total k–labelling is called the *total edge irregularity strength* of the graph G, $\mathrm{tes}(G)$. Analogously, the *total vertex irregularity strength* of G, $\mathrm{tvs}(G)$, is the minimum k for which there exists a vertex irregular total k–labelling of G. Irregular total labelling of some graph types, some bounds and some relations between δ and δ were given in their work. During the last 6 years there have been many studies on irregular total labellings of some graph types. However, there are still many open problems.

Keywords abelling, irregularity strength, total labelling, irregular total labelling

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Algorithmic and algebraic methods on Mixed Moore graphs[1]

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Both directed and undirected Moore graphs are generalized by *Mixed Moore graphs*. Mixed graphs may contain *arcs* as well as *edges* and it is known that proper mixed Moore graphs of directed degree $z \geq 1$ and undirected degree $r \geq 1$ may only exist for diameter two. In this talk we compute the traces of A^k for $k \geq 1$, where A is the adjacency matrix of any mixed Moore graph. Besides, we work on the existence of these extremal graphs in two ways. First, we develop a searching heuristic algorithm in order to find new mixed Moore graphs and recover the known ones. Second, we use the GAP library of finite groups to conclude that mixed Moore graphs of certain orders must be non–Cayley.

Keywords Mixed Moore graphs, Cayley digraphs.

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The $\otimes_h$–product: a link among labelings

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A graph labeling is an assignment of the elements of a certain set (usually the integers) to the vertices or edges, or both subject to certain conditions. Rosa introduced in 1967, the concept of graceful labeling as a way to attack the Ringel'conjecture that states that every tree T of order p decomposes K_{2p+1} into $2p+1$ copies of T, a conjecture that still remains open. Graceful labelings are the origin of the graph labeling area and many other labelings have emerged and have been studied since. Among them, we highlight harmonious labelings [4] for the number of papers related to, and we also highlight super edge-magic labelings [1] due to the links with other labelings.
Figueroa–Centeno et al. introduced in [2] the following product of digraphs. Let D be a digraph and let $\Gamma = \{F_i\}_{i=1}^m$ be a family of digraphs such that $V(F_i) = V$, for every $i \in [1, m]$. Consider any function $h : E(D) \longrightarrow \Gamma$. Then the product $D \otimes_h \Gamma$ is the digraph with vertex set $V(D) \times V$ and $((a, x), (b, y)) \in E(D \otimes_h \Gamma)$ if and only if $(a, b) \in E(D)$ and $(x, y) \in E(h(a, b))$.
The strength of the contribution (see [5–7]) lays on the use of $\otimes_h$-product not only to provide labelings of many different types of families of graphs, but also to show interesting relationships among well studied types of labelings. We are able to obtain, in this way, deep results relating different types of labelings.

Keywords $\otimes_h$-product, super edge-magic, harmonious, k-equitable, (a, d)-edge antimagic total.

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On the Characterization of PageRank Uniform Digraphs

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PageRank [2] is a method that first creates a directed graph representing web pages and the hyperlinks among them, and next assigns a relevance score to each vertex. This score is then considered for the purpose of sorting search engine results pages so that more important pages are listed first. PageRank has also been proposed for sorting the results of blog searches [1, 3].
Data managed by social network online platforms is very sensitive [5]. Due to their private content, their release for scientific analysis requires a pre-processing that eliminates the possibility of inferring private information about individuals. The required pre-processing depends on the previous structural knowledge a malicious analyst may be aware of. In [4] the authors show that the PageRank score is a structural knowledge that has to be taken into account when social network platforms provide search engines that consider this method as a criterion for sorting search results. This is the case for blogging platforms and recommendation systems.
PageRank uniform digraphs are those graphs whose vertices have all the same PageRank score. These digraphs are interesting in the scope of privacy preserving release of social network data since all their vertices are indistinguishable with respect to that score.
In this talk, we first provide a brief introduction to the PageRank algorithm. Next, we present an example showing how the PageRank score can be used to reidentify the nodes of a trivially anonymized network. Finally, we present some results about the characterization of PageRank uniform graphs and digraphs and provide a list of open problems.

Keywords PageRank, Privacy.

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The Order/Degree Optimization Problem

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In this talk we will consider directed graphs. A well-known optimization problem is the *degree/diameter problem*, which is to determine the largest order $N(d,k)$ of a digraph given maximum out-degree d and diameter at most k.

We consider a related but less well known problem, the *order/degree problem*, which is to determine the smallest diameter $K(n,d)$ of a digraph given order n and maximum out-degree d.

For the degree/diameter problem, there is a natural upper bound on the number of vertices of the digraph given maximum out-degree d and diameter at most k, namely,

$$N(d,k) = \sum_{i=0}^{k} n_i \leq 1 + d + d^2 + \cdots + d^k = \frac{d^{k+1}-1}{d-1} \tag{1}$$

The right-hand side of (1) is called the *Moore bound* and is denoted by $M(d,k)$. A digraph of (necessarily constant) out-degree d, diameter k and order equal to $M(d,k)$ is called a *Moore digraph.*

With regards to the order/degree problem, we can derive a lower bound for the diameter by performing a log operation and taking into account that for $d > 1$ and $k > 1$ the Moore bound is not attainable. Then we obtain

$$K(n,d) \geq \lceil \log_d(n(d-1)+d) \rceil - 1 \tag{2}$$

where $1 < d \leq n-1$ and $\lceil x \rceil$ is the smallest integer larger than x. The best current upper bound

can be obtained in several ways; interestingly, it is never more than one away from the lower bound. In this talk we will give an overview of the order/degree problem and present several construction techniques that produce optimal digraphs of given order and maximum out-degree. Finally, we state several related open problems.

Keywords Digraphs, Order, Degree, Diameter.

Recent advances in the Degree/Diameter Subgraph Problem in the Mesh[1]

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The Degree–Diameter Subgraph Problem (DDS) is a natural generalization of the Degree–Diameter Problem, where an undirected host graph G is given, together with two integers $\Delta, D \geq 2$, and the problem consists in finding the largest subgraph of G (in terms of the number of vertices), with maximum degree at most Δ, and diameter at most D. DDS was introduced in [1], and the case where the host graph is a k-dimensional mesh is discussed in more detail in [3]. This latter paper provides lower bounds (in the form of constructions) for the cases $k = 2, \Delta = 3$, and $k = 3, \Delta = 4$. The mesh has been revisited in [2], where the constructions of [3] have been generalized to higher dimensions. We review these results, and discuss some related open problems.

Keywords Degree/Diameter Problem, Degree/Diameter Subgraph Problem, mesh.

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Endorsement Deduction and Ranking in Social Networks[1]

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Some social networks, such as LinkedIn, allow user endorsements for specific skills. Thus, for each skill we get a directed graph where the nodes correspond to users' profiles and the arcs represent endorsement relations. From the number and quality of the endorsements received, an authority score can be assigned to each profile. We propose an authority score computation method based on the weighted PageRank algorithm [1], which takes into account the relations existing among different skills.

Keywords Social networks, PageRank, endorsement.

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MaxDDBS for digraphs

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Recently a new problem related to the Degree/Diameter Problem (DDP) for undirected graphs has been stated as: Given a connected host graph G, an upper bound for the degree d, and an upper bound for the diameter k, what is the largest subgraph S of G with maximum degree at most d and diameter at most k (MaxDDBS). The DDP is actually just a special case of the MaxDDBS, with the host graph a sufficiently large complete graph. Other important host graphs are common parallel architectures such as the mesh, the honeycomb, and the butterfly network. Upper and lower bounds for these have been given for arbitrary dimensions.

In this talk we present an oriented version of the MaxDDBS as, given the above criteria, what orientation of G gives the largest subdigraph S of this orientation with maximum out-degree d and diameter at most k. Note that the directed DDP is not a special case of the oriented MaxDDBS with the complete digraph as host digraph, but we could give a similar version of a directed MaxDDBS, where each edge is replaced by a digon. Obviously upper bounds for the order of the largest subdigraph in the oriented MaxDDBS are given by the upper bounds for the MaxDDBS. In this talk we look at the triangular mesh in two dimensions as the host digraph, so $d \leq 5$ (as the diameter is finite), and we give constructions for $d = 4, 5$ which attain the upper bound.

Keywords Digraphs, Order, Degree, Diameter.

Metric Dimension of Graphs and Digraphs: Recent Results and Open Problems

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The metric dimension problem was first introduced in 1975 by Slater [12], and independently by Harary and Melter [6] in 1976; however the problem for hypercube was studied (and solved asymptotically) much earlier in 1963 by Erdős and Rényi [4]. A set of vertices S *resolves* a graph G if every vertex is uniquely determined by its vector of distances to the vertices in S. The *metric dimension* of G is the minimum cardinality of a resolving set of G.

Garey and Johnson [5] showed that determining the metric dimension of an arbitrary graph is an NP-complete problem. Thus research in this area are then constrained towards: characterizing graphs with particular metric dimensions, determining metric dimensions of particular graphs, and constructing algorithm that best approximate metric dimensions. Until today, only graphs of order n with metric dimension 1, $n-3$, $n-2$, and $n-1$ have been characterized [2,8,11]. On the other hand, researchers have determined metric dimensions for many particular classes of graphs. In the area of constructing algorithm that best approximate metric dimensions, recently researchers have utilized integer programming [3], genetic algorithm [9], variable neighborhood search based heuristic [10], and greedy constant factor approximation algorithm [7].

Recently in 2011, Bailey and Cameron [1] established relationship between the base size of automorphism group of a graph and its metric dimension; this result then motivated researchers to study metric dimensions of distance regular graphs. There are also some results of metric dimensions of graphs resulting from graph operations.

In this talk I will present a short historical account, known techniques, recent results, and open problems in the area of metric dimension.

Keywords distance, resolving set, metric dimension.

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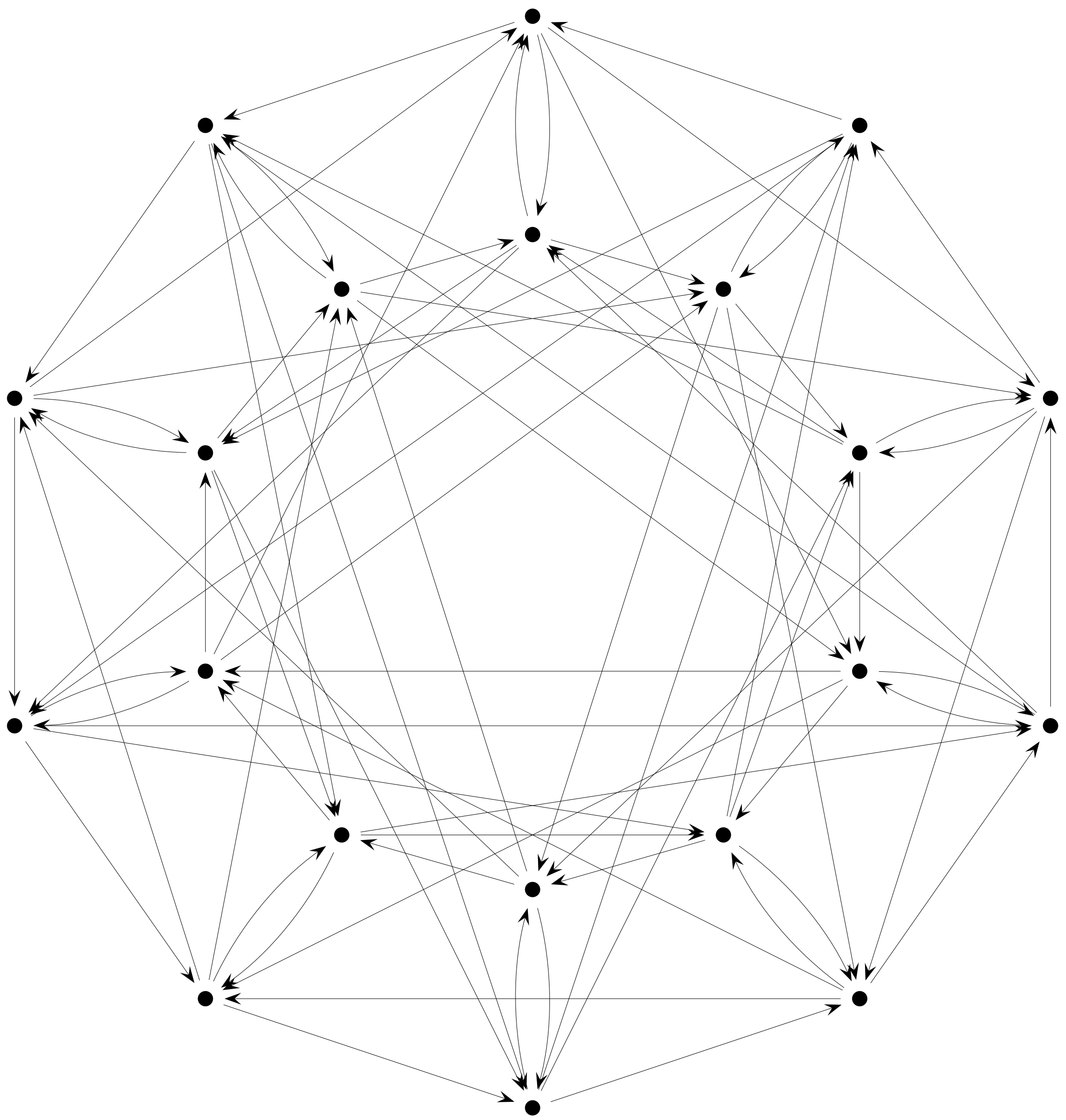

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